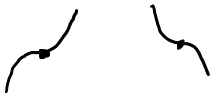
MATH 2D Prep: Local Extreme Values

Facts to Know:

For simplicity, we assume all functions in this script are continuous and differentiable everywhere.

1. Critical numbers: $x = c$ is a critical number of the function $f(x)$ if $f'(c) = 0$.

2. First derivative test: Suppose c is critical number of $f(x)$
   - If f' changes from positive(+) to negative(−), then f has a local maximum at c, 
   - If f' changes from − to +, then f has a local minimum at c, 
   - If f' is + to the left and right of c, or − to the left and right of c, then f has no local extreme value at c, 

3. Second derivative test: If $f'(c) = 0$ and
   - $f''(c) > 0$, then f has a local minimum at c, 
   - $f''(c) < 0$, then f has a local maximum at c, 
   - $f''(c) = 0$, then no conclusion.

Example:

1. Find all local extreme values of $f(x) = x^3 - 3x$.

   - Find critical numbers:

   $f'(x) = 3x^2 - 3 = 3(x^2 - 1) = 0 \Rightarrow x^2 - 1 = 0 \Rightarrow x = \pm 1$

   2 crit. numbers: 1 and −1

   - Method 1: First derivative test:

Inteval	$f'(x)$	$f(x)$
$x < -1$	+	↑
$-1 < x < 1$	−	↓
$x > 1$	+	↑

   $f(x)$ has:
   local max. $f(-1) = 2$ at $x = -1$
   local min $f(1) = -2$ at $x = 1$

   - Method 2: Second derivative test:

   $f''(x) = 3 \cdot 2x - 0 = 6x$

x	$f''(x)$	$f(x)$
1	6 (+)	local min
−1	−6 (−)	local max

   f has:
   local min $f(1) = -2$ at $x = 1$ and
   local max value $f(-1) = 2$ at $x = -1$